International Journal of Multidisciplinary and Scientific Emerging ResearcH (IJMSERH)

| ISSN: 2349-6037 | www.ijmserh.com ||Impact Factor: 8.325||Quarterly, Peer Reviewed & Refereed Journal|

|| Volume 11, Issue 3, July 2023 ||

Weighted Fuzzy Similarity Measure Based on Tangent Function and its Application to Medical Diagnosis

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ABSTRACT: In this paper, the weighted tangent similarity measure of fuzzy sets is proposed and its properties are studied. The concept of the weighted tangent similarity measure of fuzzy sets is a decision making tool which is characterized by the degree of membership function, degree of non-membership function (sum of this two components is equal to one). Finally, using this proposed method, an application on medical diagnosis is given for the applicability of the proposed approach.

KEYWORDS: Tangent similarity measure, weighted tangent similarity measure, fuzzy sets, decision making, and medical diagnosis.

I. INTRODUCTION

It is recognized that uncertainty plays an important role in modelling real world problems. Researchers recognize the need to bridge the gap between mathematical models and uncertainty and their empirical interpretations. The reflection of this gap can be found in problems of operations research, mathematics, biological, cognitive, and social sciences as well as modern technology, medicine and other applied sciences. The need to bridge the gap between a mathematical model and experience is well addressed by Max Black [1] in 1937. Zadeh [2] expressed the same need in 1962. In 1965, Zadeh [3] proposed the new paradigm of mathematics based on the very concept of fuzzy sets. When the new paradigm was proposed [3], the usual process of a paradigm shift [4]] begins. For details of scientific paradigm you may consult the highly influential book namely, "The Structure Of Scientific Revolutions" "authored by Kuhn [4]. The paradigm shift is still continuing. It is reflected in the challenges thrown by the theory of fuzzy sets to the very foundation of science i.e. the Aristotelian two-valued logic. The new paradigm has a greater capability to deal with human decision making, machine intelligence, etc. The concept of Fuzzy set [3] generalizes the Cantor set discovered by Smith [5] in 1874 and introduced by German mathematician Cantor [6] in 1883. In fuzzy set theory, membership and non-membership degrees are complementary, i.e., the sum of membership and non-membership degrees of an element belonging in a fuzzy set is equal to one. The fuzzy set theory facilitates to solve various real world problems involving partially unknown information.

Literature review reflects that the studies on fuzzy similarity measures are mostly theoretic [7], [8], [9], [10], [11]. Fuzzy similarity measures are applied to image processing. [12], [13], fuzzy reasoning. [14], medical diagnosis [15], etc. Kakati has established a new similarity measure [16] for fuzzy sets using the extended definition of complementation [17] based on reference function.. Kakati has proved the validity [18] of the new similarity measure [16] with the help of traditional Hamming Distance and Euclidean Distance measures and applied it to medical reasoning.

In this paper the authors have proposed a new similarity measure of fuzzy sets namely; weighted fuzzy similarity measure based on tangent function and studied its basic properties and applied it to medical diagnosis. The rest of the paper is organized as follow:

Section II presents the concepts of FSs, tangent similarity measures and weighted tangent similarity measures for fuzzy sets. Section III presents decision making methodology based on weighted tangent similarity measures. Section IV is devotes to present an example on medical diagnosis using the proposed approach. Finally, the conclusion of the paper and scope of future work are presented in Section V.

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II. MATHEMATICAL PRELIMINARIES

Fuzzy set: In 1965, Zadeh [3] introduced the concept of fuzzy sets as a mathematical form for representing impreciseness.

Definition 2.1: Fuzzy set: A fuzzy set A in a universe of discourse X is defined as the following set of pairs $A=\{(x,\mu_A(x)): x\in X\}$. Here, $\mu_A(x): x\to[0,1]$ is a mapping called the membership value of x ∈ X in a fuzzy set A

Definition 2.2: The value of $\nu_A(x)=1-\mu_A(x)$ is called the degree of non membership of the element x ∈ X to the fuzzy set.

Definition 2.3: **Fuzzy Number**: A fuzzy number is an extension of a regular number such that it does not refer to one single value but related to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. Thus a fuzzy number is a normal fuzzy set and convex set.

Definition 2.4: Hamming distance between two fuzzy sets A and B is defined as

$$H(A,B)=\frac{1}{2}\sum_{i=1}^{n}\left|\mu_A(x)-\mu_B(x)\right|+\left|\nu_A(x)-\nu_B(x)\right|$$

Definition 2.5: Tangent similarity measure for fuzzy sets

The authors propose the tangent similarity measure for fuzzy sets in the following way.

Let $P=(\mu_P(x_i),\nu_P(x_i))$ and $Q=(\mu_Q(x_i),\nu_Q(x_i))$ be two fuzzy numbers. Now tangent similarity function ($T_{FS}(P,Q)$) which measures the similarity between two vectors P and Q based only on the direction, ignoring the impact of the distance between them can be presented as:

$$T_{FS}(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left\langle 1-\tan\left(\frac{\pi\left(\left|\mu_P(x_i)-\mu_Q(x_i)\right|+\left|\nu_P(x_i)-\nu_Q(x_i)\right|\right)}{8}\right)\right\rangle \quad (1)$$

Proposition 1. The defined tangent similarity measure $T_{FS}(P,Q)$ between two fuzzy numbers P and Q satisfies the following properties:

1. $0\le T_{FS}(P,Q)\le 1$
2. $T_{FS}(P,Q)=1$ if and only if $P=Q$
3. $T_{FS}(P,Q)=T_{FS}(Q,P)$
4. If R is a FS in X and $P\subset Q\subset R$ then $T_{FS}(P,R)\le T_{FS}(P,Q)$ and $T_{FS}(P,R)\le T_{FS}(Q,R)$

Proofs:

(1)

As the membership, non-membership function of the fuzzy set are in [0, 1] and the value of the tangent function are within [0 ,1], the similarity measure based on tangent function also is within [0,1]. Hence $0\le T_{FS}(P,Q)\le 1$

(2)

For any two fuzzy sets P and Q if $P=Q$ this implies $\mu_P(x_i)=\mu_Q(x_i), \nu_P(x_i)=\nu_P(x_i)$. Hence $\left|\mu_P(x_i)-\mu_Q(x_i)\right|=0$, $\left|\nu_P(x_i)-\nu_Q(x_i)\right|=0$, Thus $T_{FS}(P,Q)=1$

Conversely,

If $T_{FS}(P,Q)=1$ then $\left|\mu_P(x_i)-\mu_Q(x_i)\right|=0$, $\left|\nu_P(x_i)-\nu_Q(x_i)\right|=0$, since tan(0) = 0.

So we can write $\mu_P(x_i)=\mu_Q(x_i)$, $\nu_P(x_i)=\nu_Q(x_i)$. Hence P = Q.

(3)

This proof is obvious.

(4)

If $P\subset Q\subset R$ then $T_P(x_i)\le T_Q(x_i)\le T_R(x_i)$, and $F_P(x_i)\ge F_Q(x_i)\ge F_R(x_i)$ for $x_i\in X$.

Now we have the following inequalities:

$\left|T_P(x_i)-T_Q(x_i)\right|\le\left|T_P(x_i)-T_R(x_i)\right|$, $\left|T_Q(x_i)-T_R(x_i)\right|\le\left|T_P(x_i)-T_R(x_i)\right|$;

$\left|F_P(x_i)-F_Q(x_i)\right|\le\left|F_P(x_i)-F_R(x_i)\right|$, $\left|F_Q(x_i)-F_R(x_i)\right|\le\left|F_P(x_i)-F_R(x_i)\right|$.

Thus $T_{FS}(P,R)\le T_{FS}(P,Q)$ and $T_{FS}(P,R)\le T_{FS}(Q,R)$. Since tangent function is increasing in the interval $\left[0,\frac{\pi}{4}\right]$.

| ISSN: 2349-6037 | www.ijmserh.com ||Impact Factor: 8.325||Quarterly, Peer Reviewed & Refereed Journal|

|| Volume 11, Issue 3, July 2023 ||

Definition 2.6: Weighted tangent similarity measure for fuzzy sets

The authors propose weighted tangent similarity measure for fuzzy sets in the following way.

Let $P = (\mu_P(x_i), \nu_P(x_i))$ and $Q = (\mu_Q(x_i), \nu_Q(x_i))$ be two fuzzy numbers. Now we present weighted tangent similarity function ($T_{WFS}(P, Q)$) which measures the similarity between two vectors P and Q based only on the direction, ignoring the impact of the distance between them can be presented as:

$$T_{WFS}(P, Q) = \frac{1}{n}\sum_{i=1}^{n} w_i \left\langle 1 - \tan\left(\frac{\pi\left(\left|\mu_P(x_i) - \mu_Q(x_i)\right| + \left|\nu_P(x_i) - \nu_Q(x_i)\right|\right)}{8} \right) \right\rangle \quad (2)$$

Where, $0 \le w_i \le 1$, i = 1, 2, …, n; and $\sum_{i=1}^{n} w_i = 1$. Definition **2.6** converts to definition **2.5** if $w_i = \frac{1}{n}$

Proposition 2. The defined tangent similarity measure $T_{WFS}(P, Q)$ between two fuzzy numbers P and Q satisfies the following properties:

5. $0 \le T_{WFS}(P, Q) \le 1$
6. $T_{WFS}(P, Q) = 1$ if and only if $P = Q$
7. $T_{WFS}(P, Q) = T_{FS}(Q, P)$
8. If R is a FS in X and $P \subset Q \subset R$ then $T_{WFS}(P, R) \le T_{WFS}(P, Q)$ and $T_{WFS}(P, R) \le T_{WFS}(Q, R)$

Proofs:

(5)

As the membership, non-membership function of the fuzzy set are in [0, 1] and the value of the tangent function are within [0 ,1], the similarity measure based on tangent function also is within [0,1]. Hence $0 \le T_{WFS}(P, Q) \le 1$

(6)

For any two fuzzy sets P and Q if $P = Q$ this implies $\mu_P(x_i) = \mu_Q(x_i)$, $\nu_P(x_i) = \nu_Q(x_i)$. Hence $\left|\mu_P(x_i) - \mu_Q(x_i)\right| = 0$, $\left|\nu_P(x_i) - \nu_Q(x_i)\right| = 0$, Thus $T_{WFS}(P, Q) = 1$

Conversely,

If $T_{WFS}(P, Q) = 1$ then $\left|\mu_P(x_i) - \mu_Q(x_i)\right| = 0$, $\left|\nu_P(x_i) - \nu_Q(x_i)\right| = 0$, since tan(0) = 0.

So we can write $\mu_P(x_i) = \mu_Q(x_i)$, $\nu_P(x_i) = \nu_Q(x_i)$. Hence P = Q.

(7)

This proof is obvious.

(8)

If $P \subset Q \subset R$ then $T_P(x_i) \le T_Q(x_i) \le T_R(x_i)$, and $F_P(x_i) \ge F_Q(x_i) \ge F_R(x_i)$ for $x_i \in X$.

Now we have the following inequalities:

$\left|T_P(x_i) - T_Q(x_i)\right| \le \left|T_P(x_i) - T_R(x_i)\right|$, $\left|T_Q(x_i) - T_R(x_i)\right| \le \left|T_P(x_i) - T_R(x_i)\right|$;

$\left|F_P(x_i) - F_Q(x_i)\right| \le \left|F_P(x_i) - F_R(x_i)\right|$, $\left|F_Q(x_i) - F_R(x_i)\right| \le \left|F_P(x_i) - F_R(x_i)\right|$.

Thus $T_{WFS}(P, R) \le T_{WFS}(P, Q)$ and $T_{WFS}(P, R) \le T_{WFS}(Q, R)$. Since tangent function is increasing in the interval $\left[0, \frac{\pi}{4}\right]$.

III. FUZZYY DECISION MAKING BASED ON TANGENT FUNCTION

Let $A_1, A_2, ..., A_m$ be a discrete set of candidates, $C_1, C_2, ..., C_n$ be the set of criteria of each candidate, and $D_1, D_2, ..., D_k$ are the alternatives of each candidate. The decision-maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performances of candidates A_i (i = 1, 2,..., m) against the criteria C_j (j = 1, 2, ..., n). The values associated with the alternatives for MADM problem can be presented in the following two decision matrices (see Table 1 and Table 2).

| ISSN: 2349-6037 | www.ijmserh.com ||Impact Factor: 8.325||Quarterly, Peer Reviewed & Refereed Journal|

|| Volume 11, Issue 3, July 2023 ||

Table 1: The relation between candidates and attributes

	C_1	C_2	...	C_n
A_1	$[AC]_{11}$	$[AC]_{12}$	...	$[AC]_{1n}$
A_2	$[AC]_{21}$	$[AC]_{22}$	...	$[AC]_{2n}$
...	...	...	...	...
A_m	$[AC]_{m1}$	$[AC]_{m2}$	...	$[AC]_{mn}$

Table 2: The relation between attributes and alternatives

	D_1	D_2	...	D_k
C_1	$[CD]_{11}$	$[CD]_{12}$	...	$[CD]_{1k}$
C_2	$[CD]_{21}$	$[CD]_{22}$	...	$[CD]_{2k}$
...	...	...	...	...
C_n	$[CD]_{n1}$	$[CD]_{n2}$	...	$[CD]_{nk}$

Here $[AC]_{ij}$ and $[CD]_{jk}$ and are all fuzzy numbers.
The steps of decision making corresponding to fuzzy number based on tangent function are presented as following steps.

Step 1: Determination of the relation between candidates and attributes:
Each candidate A_i (i = 1, 2, ..., m) having the attribute C_j (j = 1, 2, ..., n). The corresponding relational values between candidates and their attributes are presented in terms of fuzzy numbers as follows (see Table 3):

Table 3: Relation between candidates and attributes in terms of fuzzy numbers

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, F_{11}\rangle$	$\langle T_{12}, F_{12}\rangle$	...	$\langle T_{1n}, F_{1n}\rangle$
A_2	$\langle T_{21}, F_{21}\rangle$	$\langle T_{22}, F_{22}\rangle$	...	$\langle T_{2n}, F_{2n}\rangle$
...	...	...	...	...
A_m	$\langle T_{m1}, F_{m1}\rangle$	$\langle T_{m2}, F_{m2}\rangle$	...	$\langle T_{mn}, F_{mn}\rangle$

Step 2: Determination of the relation between attributes and alternatives:
The relation between attributes C_i (i = 1, 2, ..., n) and alternatives D_t (t = 1, 2, ..., k) is presented as follows (see Table 4):

Table4: The relation between attributes and alternatives in terms of fuzzy numbers

	D_1	D_2	...	D_k
C_1	$\langle T_{11}, F_{11}\rangle$	$\langle T_{12}, F_{12}\rangle$	...	$\langle T_{1k}, F_{1k}\rangle$
C_2	$\langle T_{21}, F_{21}\rangle$	$\langle T_{22}, F_{22}\rangle$	...	$\langle T_{2k}, F_{2k}\rangle$
...	...	...	...	...
C_n	$\langle T_{n1}, F_{n1}\rangle$	$\langle T_{n2}, F_{n2}\rangle$	...	$\langle T_{nk}, F_{nk}\rangle$

Step 3: Determination of the criteria weight structure of similarity measure:
In the diagnosis process, decision maker may often encounter with unknown criteria weights. It may happen that the importance of the criteria is different. Therefore it is necessary to determine reasonable criteria weight for similarity measures.

Step 4: Determination of the co-relation measure between two relations:
Determine the correlation measure between the Table 3 and the Table 4 using $T_{WFS}(P, Q)$ (from the equation 2).

Step 5: Ranking the alternatives:
Ranking the alternatives corresponding to each candidate is prepared as the descending order of correlation measures. Highest value indicates the best alternative for corresponding candidate.

Step 6: End

| ISSN: 2349-6037 | www.ijmserh.com ||Impact Factor: 8.325||Quarterly, Peer Reviewed & Refereed Journal|

|| Volume 11, Issue 3, July 2023 ||

IV. NUMERICAL EXAMPLE ON MEDICAL DIAGNOSIS BASED ON TANGENT FUNCTION

Let us consider a numerical example on medical diagnosis. Medical diagnosis consists of a large amount of uncertainties and increased volume of information available to physicians from new medical technologies. The process of classifying different set of symptoms is under a single name of a disease. The proposed similarity measure among the patients Vs symptoms and symptoms Vs diseases gives proper medical diagnosis. The main feature of this proposed method is that it considers membership, non-membership degree by taking one time inspection for diagnosis.

Now, an example of a medical diagnosis will be presented. Example: Let P = {P_1, P_2, P_3, P_4} be a set of patients, D = {Viral Fever, Malaria, Typhoid, Stomach problem, Chest problem} be a set of diseases and S= {Temperature, Headache, Stomach pain, cough, Throat pain.} be a set of symptoms. Our solution is to examine the patients which in turn gives arise to membership and non-membership function for each patient.

Step 1: Determination of the relation between candidates and attributes:
Four patients (P_1, P_2, P_3, and P_4) have the symptoms temperature, Headache, Stomach pain, cough, and Throat pain. They feel illness. With the help of expert assessments, we tabulate the relational values between patients and their symptoms as follows (see Table 5).

Table 5: (Relation-1) - The relation between Patient and Symptoms

Relation-1	Temperature	Headache	Stomach pain	cough	Throat pain
P_1	(0.8, 0.2)	(0.6, 0.4)	(0.5, 0.5)	(0.6, 0.4)	(0.4,0.6)
P_2	(0.8, 0.2)	(0.4, 0.6)	(0.6, 0.4)	(0.7, 0.3)	(0.7, 0.3)
P_3	(0.7, 0.3)	(0.7, 0.3)	(0.6, 0.4)	(0.5, 0.5)	(0.5, 0.5)
P_4	(06, 0.4)	(0.5, 0.5)	(0.7, 0.3)	(0.8, 0.2)	(0.7, 0.3)

Step 2: Determination of the relation between attributes and alternatives:
Every disease has some symptoms. There are some diseases whose symptoms are more or less same. So, in medical diagnosis, there is a confusion to detect exact disease of a patient. That's why, it is important to set up the relations between symptoms and diseases in uncertain environment. Here, the relations are presented in fuzzy numbers as follows (see Table 6).

Table 6: (Relation-2) -The relation among Symptoms and Diseases

Relation-2	Viral Fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.7, 0.3)	(0.6, 0.4)	(0.4, 0.6)	(0.4, 0.6)	(0.6, 0.4)
Headache	(0.5,0.5)	(0.5, 0.5)	(0.3, 0.7)	(0.2, 0.8)	(0.3, 0.7)
Stomach pain	(0.3, 0.7)	(0.1, 0.9)	(0.1, 0.9)	(0.9, 0.1)	(0.2, 0.8)
Cough	(0.8, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.4, 0.6)	(0.8, 0.2)
Throat pain	(0.6, 0.4)	(0.3, 0.7)	(0.3, 0.7)	(0.2, 0.8)	(0.6, 0.4)

Step 3: Determination of the weight structure of each similarity measure:
Weight structure of each criterion for proposed similarity measure is determined by expert (doctor/ medical practitioner) as follows:
$w_1 = 0.225$, $w_2 = 0.195$, $w_3 = 0.200$, $w_4 = 0.190$, $w_5 = 0.190$

Step 4: Determination of the co-relation measure between two relations:
Using equation 2 (tangent function) we calculate correlation measures between Relation-1 and Relation-2 as follows (see Table 7).

Table 7: The Correlation Measure between Relation-1 and Relation-2

Weighted Tangent similarity measure	Viral Fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	0.8751	**0.8841**	0.7625	0.7384	0.8093
P_2	**0.8890**	0.8044	0.7437	0.7428	0.8541
P_3	**0.8605**	0.8080	0.7566	0.7566	0.7934
P_4	**0.9024**	0.8214	0.7454	0.7454	0.8713

Step 5: Ranking the alternatives:

| ISSN: 2349-6037 | www.ijmserh.com ||Impact Factor: 8.325||Quarterly, Peer Reviewed & Refereed Journal|

|| Volume 11, Issue 3, July 2023 ||

The highest correlation measure from the Table 7 gives proper medical diagnosis. Therefore, patient P_1 suffers from Malaria, P_2 suffers from Viral fever, P_3 suffers from Viral fever and P_4 suffers from viral fever.

V. CONCLUSION

In this paper, we have proposed a tangent similarity measure approach of fuzzy sets and proved some of their basic properties. We have presented an application of weighted tangent similarity measure of fuzzy sets in medical diagnosis problem. In the future work, we will extend this tangent similarity measure to fuzzy multi sets.

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